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Measurement of the Dynamic Elastic Modulus of Short Staple Fibers

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Synopsis

A description is given of the equipment and measurement techniques which have been developed for the determination of the dynamic elastic modulus of staple fibers of length 25 mm or less. The dynamic elastic modulus, often called the sonic modulus, is determined by means of the pulse propagation method with the aid of a specially designed constant fiber-strain device. The modulus values are obtained for different fiber strain levels and are used to construct the dynamic stress–strain curve of the fibers. Results obtained for cotton, jute, and wool fibers are presented and discussed. In general, two points of inflexion on the constructed fiber stress–strain curves are determined without ambiguity, by the maximum and minimum values of the fiber dynamic modulus. The region between these two inflexion points is interpreted as the yield region of the fiber.

INTRODUCTION

In textile processing, fibers are subjected to ever-increasing rates of stressing as the speeds of moving parts in modern high production machinery are increased. It is of considerable importance to know the capacity of the fiber to resist these stresses and also its capacity to cope with the stresses and strains in subsequent stages of processing and in the actual service life of the finished product. Properties related to the fatigue or yield point of the fibers are particularly relevant to these problems.

Since the elastic modulus is the first derivative of the stress–strain relationship, it follows that direct measurement of the modulus–strain or modulus–stress relationship should provide a more sensitive and critical means of analysing the dynamic stress–strain behavior of the specimen than direct measurement of the conventional stress–strain curve itself.

However, elastic modulus is often obtained from the slope of the experimentally obtained load–extension curve. The measured load–extension curve is converted to the stress–strain curve, where specimen dimensions such as the specimen length, cross-sectional area, or linear density are introduced. When the modulus is obtained as the slope of the curve or as a quotient of these quantities, further experimental or computational errors are introduced.

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The elastic modulus may be measured directly by dynamically simulating the force–stretching conditions in the specimen. The importance of the dynamic elastic modulus in the study of cotton fibers is evident from the intrinsic nature of the elastic modulus and is demonstrated by such relationships as dynamic modulus versus birefringence,[1,2] birefringence versus fiber maturity,[3] modulus versus molecular orientation,[4] etc. Similar observations have been reported for other fibers.[5–9]

Measurement of the dynamic stress–strain properties of crystals, stiff whiskers, filaments, yarns, cords, films, paper, fabrics, and rubber pieces have been widely reported in the literature, but very few data on short, flexible, fine, single fibers such as cotton and wool are available.

A number of different measurement techniques may be considered for the determination of the dynamic elastic modulus of single filaments. However, the restrictions imposed by the dimensions and the geometry of short fibers present considerable experimental difficulties.

In the present work, a set of criteria was established incorporating the features of the short fibers. An instrument and associated experimental techniques were developed to satisfy the set criteria. The measurement techniques appearing in the literature are briefly discussed and compared, and the set criteria are explained against this background.

A frequency of 1 Hz in tensile tests may be conveniently set as the borderline dividing static (or quasi-static) and dynamic test conditions.[10] In the following review, this convention is adopted and only those techniques employing frequencies not less than 1 Hz are included.

EXPERIMENTAL TECHNIQUES

Direct Force-Stretching Methods

In these methods, the modulus is calculated from the observed stress–strain relationship.

Cyclic Tests. Wakeham and Honold[11] used a cam-generated mechanical vibration of 1 Hz to apply a cyclic load to the fiber specimen and obtained the elastic modulus of single cotton fibers. In order to determine the modulus, measured values of the original specimen length and cross-sectional area were used. The authors do not present any stress–strain curves for cotton or other short fibers.

Impact Loading Tests. Work related to the impact strength of short staple fibers has been reported by Balls[12,13] for cotton and Lang[14] for wool. Both employed a pendulum to break the specimen in bundles. Since the impact strength is a terminal property, these tests cannot provide the dynamic stress–strain relationship on a continuous basis.

Indirect or Simulated Force-Stretching Methods

These methods may be used for the direct determination of dynamic elastic modulus.

Vibration Techniques. These techniques may be further classified as free vibration or forced vibration methods.

(a) *Free Vibration Methods.* Asmussen and Andersen[15] and Andersen[16] used a form of free vibration method in which a compound system (consisting of a spring vibrator of known stiffness, a loading weight, and a fiber specimen of known dimensions) was set into vibration by an electromagnetic impulse of 15 to 30 Hz. By measuring the periods of the vibration with and without the specimen, the dynamic elastic modulus of the short specimen was determined. The specimen, cotton of American origin, 15–30 mm in length, was held between two small pairs of tweezers. Tension in the fiber and the relative humidity were controlled during the measurements.

Ballou and Smith[5] suspended a loaded specimen and set it into vibration. The frequency was determined with the aid of a photocell. Guthrie et al.[17] measured the bending modulus of fibers into the form of a cantilever and found the dynamic modulus to be constant for frequencies between 40 Hz and 7 kHz.

Clayton and Peirce[18] used a double pendulum to measure the flexural rigidity of cotton fibers and obtained the dynamic bending modulus of soda-boiled Texas cotton. Similar methods have been used by others,[19,20] but none of these methods would appear suitable for measurements of the dynamic tensile modulus of short fibers.

(b) *Forced Vibration Methods.* These may be either resonant or nonresonant methods.

(i) Resonant Forced Vibration Methods. The specimen usually takes the form of a cantilever or is connected to a metallic cantilever. The cantilever is driven by an oscillator. In the variant of the method called the "vibrating reed technique," the specimen undergoes a flexural vibration, and the dynamic bending modulus is obtained. A number of different versions of this method have been used by Horio et al.[21] Lochner,[22] Lincoln,[23] and Kärrholm and Schröder[24] have reported measurements of the dynamic modulus of wool fibers.

Longitudinal vibration may be imposed on the specimen by means of a cantilever vibrator. Weyland[25] actually used a strained specimen attached to the free end of a metallic cantilever. The cantilever was driven at a given frequency of about 1 kHz, and its natural frequency was measured in order to determine the dynamic elastic modulus as a function of the fiber strain. The author gave curves for $1/E$ versus $\ln(1 + \epsilon)$, where E is the dynamic modulus and ϵ is the fiber strain, for rayon and nylon 6. Some measured values of the modulus for cotton, wool, ramie, and other fibers, at zero fiber strain, were listed by the author.

Inobe[26] used a modified cantilever–specimen system in which the load in the specimen was controlled. Results were reported of measurements on filaments and also on wool fibers. Another type of apparatus, known as a "stretch vibrometer," has been used by many authors.[27–30] Lyons[27] measured the dynamic modulus of ramie and other fibers, but no other author using this method reported successful measurements on short fibers.

(ii) Nonresonant Forced Vibration Methods. In this case, the specimen undergoes an alternating extensional strain in a frequency range such that the length of the fiber is short compared to the wavelength of the stress waves in the fiber. An apparatus, called Vibron or Rheovibron, was developed by Takayanagi[31] and vibrates the specimen at 3.5 Hz or 100 Hz; the stress and strain are measured by unbonded strain gauges. The apparatus accepts specimens of length between 40 and 60 mm.

Pinnock and Ward[32] used a higher frequency of 2 kHz in their modified apparatus. Again, the response of the fiber material to the imposed alternating strain was closely followed and measured by strain gauges.

Acoustic Techniques. A general feature of these techniques is that the velocity of propagation, C, of sound waves in the specimen is measured by a suitable method. The velocity C is then used to obtain the dynamic elastic modulus E_{dyn} as follows:

$$E_{\text{dyn}}\ (\text{dyn/cm}^2) = C^2 \cdot \rho$$

where ρ is the fiber density (g/cm^3) and C is the velocity (cm/sec), or

$$E_{\text{dyn}}\ (\text{Newton/tex}) = C^2$$

where C is the velocity in km/sec.

The acoustic methods may be classified according to whether the sound wave is propagated as a continuous wave or in pulse form.

(a) *Wave Propagation Methods* (i) Wave Transmission. The frequency of vibration is usually fixed and the distance between the transmitting and receiving transducers along the specimen is varied. A standing wave of definite frequency is set up in the specimen and the half-wavelength determined. The velocity C of the waves is obtained as the product of the wavelength λ and the frequency ν:

$$C = \lambda \cdot \nu.$$

The work reported by Meyer and Lotmar[33] employed this principle. The specimen and a standard fiber of known modulus were excited by longitudinal friction. The velocity of propagation of the waves in the specimen was obtained from the length ratio when the frequencies in both the specimen and the standard fiber were matched.

Applying the same principle, but with considerable refinements, Ballou and Silverman[34] obtained values for viscose and nylon filaments as a function of percentage elongation. Similar methods have been used by de Vries,[1] Ballou and Smith,[5] Nolle,[20] Hillier,[35] Witte et al.,[36] Hamburger,[37] and Fujino et al.[38]

(ii) Longitudinal Wave Resonance. Another method employing wave propagation is the method of longitudinal wave resonance. While the specimen length L is kept constant, the frequency of the oscillator is gradually varied. The resonance is observed at a specific frequency ν_r which can be detected by suitable means. The propagation velocity C of the waves in the specimen may be obtained from the following expression:

$$C = 2L\nu_r/n$$

where $n = 1, 3, 5, \ldots$. Nolle[20] and Fujino et al.[38] also used this method, but the latter combined the two wave propagation methods in one apparatus.

(b) *Pulse Propagation Methods.* A train of short pulses of high frequency oscillations are transmitted through the specimen, and the transit time of the pulses along the specimen is determined by a timing device. Hamburger[37] used a combination of a pulse generator and a number of frequency dividers to supply pulses at suitable intervals. When a piezoelectric crystal acting as the transmitter is excited by the individual pulses, the crystal vibrates at its resonant frequency of 10 kHz. This mechanical vibration is introduced into the specimen through a pressure contact between the crystal and the specimen. The vibration is then intercepted in transit by another identical crystal at a distance along the specimen, again the crystal–specimen contact being achieved by pressure between them.

In the pulse propagation method, the propagation velocity C in the specimen is obtained by dividing the crystal separation L by the transit time.

Hamburger obtained the relationship between the dynamic elastic modulus and extension for viscose, nylon, and other fibers, but his use of the minimum crystal separation distance of 7 in. automatically precluded any possibility of determining the modulus for short fibers. However, he recommended an investigation of the viscoelastic properties of natural fibers, especially staple fibers of short length. With certain improvements and modifications, the apparatus used by Hamburger and patented by Rich[39] is now commercially available as the "Dynamic Modulus Tester PPM-5R" through H. M. Morgan Co. Inc.

Chaikin and Chamberlain[40] also employed a similar method using the same principle. A crystal of Rochelle salt with resonant frequency of about 100 kHz and a condenser microphone were used as transmitter and receiver, respectively. Transducer–specimen contact was achieved by actually tieing the fiber specimen to the hooks of the transducers by means of self-tightening knots. Using the method of least squares, the authors took great care to avoid errors in determining the length–time slope. Values of the dynamic elastic modulus of wool fibers were given.

Fujino et al.[38] used a similar arrangement, but their specimens were limited to continuous filaments. King and Kruger[41] used a higher frequency of 0.5 MHz in their pulse propagation arrangement. The fiber specimen was stretched over two vertical Perspex supports and fixed into position by means of adhesive tape. The authors gave values of the dynamic elastic modulus of mohair only.

TABLE I
Dynamic Elastic Modulus of Some Natural Fibers

Fiber	Dynamic elastic modulus		Frequency	Source
	10^{10} dyn/cm^2	Newton/tex[a]		
Cotton	14–27 bending	9.21–17.76 bending	1.5–1.8 Hz	Clayton and Peirce[18]
Cotton	3.5–10.0	2.30–6.58	15–30 Hz	Asmussen and Andersen[15]
Cotton	3.0–17.0	1.97–11.18	25 Hz	Andersen[16]
Cotton	7.3–11.3	4.80–7.43	1 Hz	Wakeham and Honold[11]
Cotton	16.0	10.52	1 kHz	Weyland[25]
Flax	80–110	51.94–71.43	7 kHz	Meyer and Lotmar[33]
Hemp	35–70	22.73–45.45	7 kHz	same
Jute	66	42.85	7 kHz	same
Ramie	19–60	12.34–38.96	7 kHz	same
Ramie	31.9	20.71	180 Hz	Lyons[27]
Ramie	66	42.85	1 kHz	Weyland[25]
Wool	8.1–9.2	6.18–7.02	100 kHz	Chaikin and Chamberlain[40]
Wool	7.47 bending	5.70 bending	up to 45 Hz	Lochner[22]
Wool	7.8–9.2 bending	5.95–7.02 bending	1.6–60 Hz	Lincoln[23]
Wool	6.09–6.23 bending	4.65–4.75 bending	under 100 Hz	Kärrholm and Schröder[24]
Wool	2.0	1.52	237 Hz	Inobe[26]
Wool	4.4 bending	3.36 bending	0.1–2 kHz	Guthrie et al.[17]
Wool	6.5	4.93	1 kHz	Weyland[25]
Mohair	1.90–3.57	1.45–2.72	0.5 MHz	King and Kruger[41]

[a] Converted from dyn/cm^2 using suitable values of fiber density.

In the pulse propagation methods, the only measurement required is the time taken for a single pulse to travel a measured or known distance in the specimen. It follows that one distinct advantage of the pulse propagation methods (over any other method) is the fact that the dynamic modulus obtained by this method is independent of the dimensions of the specimen. Considering the irregular cross-sectional and convoluted shape of cotton fibers and the crimps in fine wool fibers, the errors originating from measurements of fiber dimensions may easily outweigh the magnitude of variations in the material properties.

Table I gives a summary of the values of the dynamic elastic modulus obtained by various workers for some natural fibers.

DESIGN OF DYNAMIC MODULUS TESTER

Criteria for a Satisfactory Method

The following criteria were applied to the design of an instrument for measurement of the dynamic elastic modulus of short fibers.

1. The measure of modulus should be independent of the cross-sectional area and the shape of the fiber specimen. Only the acoustic techniques qualify according to this criterion.

2. The apparatus must be capable of determining the dynamic elastic modulus of short fibers of length 25 mm or less. With all wave propagation methods, the measurement principle necessitates that the specimen must contain more than one wavelength of the vibration, thereby imposing the difficult problem of accommodating a short fiber. On the other hand, the requirement for all vibration techniques is that the specimen length must be less than the wavelength concerned, a distinct advantage in this case.

3. Since the dynamic elastic modulus of many polymeric materials varies markedly during a force-stretching process, the method should permit continuous determination of the dynamic modulus on a single specimen over the entire straining process leading to breakage. In this way, the modulus–strain relationship may be obtained on a single specimen.

4. The frequency to be employed must be high enough to eliminate the effects of primary and secondary creep and also to simulate dynamic conditions.

Taking the above criteria into account, it is clear that the acoustic pulse propagation technique appears the most suitable principle of measurement for the present application. The obvious limitation of the pulse propagation method is that the attenuation, and therefore the loss modulus, cannot be obtained.

Successful application of the pulse propagation method requires proper consideration of such problems as: the effectiveness of the transducer–specimen contact; the violin bow effect in the specimen; the effect of the transducer separation; accuracy and precision of the distance measurement along the specimen; the specification and standardization of the strain level in the specimen. These problems are considered in the description of the apparatus developed and used in the present work.

The technique employed should be free from serious sources of error, such as end effects resulting from reflection of waves at the end fixtures holding the specimen and phase shifts which may occur at unpredictable periods during the test.

Description of the Apparatus

The Dynamic Modulus Tester PPM-5R manufactured and supplied by H. M. Morgan Co. Inc.[42] was found to have many desirable features meeting the above-listed criteria. Short pulses about 0.5 microsecond long are generated at the rate of 130 kHz, and the rate is lowered by successive frequency dividers to provide the transmitting crystal with the pulses at the rate of 160 Hz, or at intervals of 6250 microseconds between pulses.[39] In most textile fibers, a pulse of 10 kHz travels with a velocity in the range of 1 to 5 km/sec, i.e. it takes between 5 to 25 microseconds to travel a distance of 25 mm along the specimen. Therefore, this rate of pulse transmission is sufficiently low to permit the amplitude of each pulse to decay to zero before the initiation of the next pulse. As a result, the time of arrival of each pulse at the receiving crystal is individually measured without being influenced by end effects of the specimen holders.

Another important feature of this tester is the movement of the transmitting transducer which permits automatic recording of the relationship between the pulse transit time and the distance scanned along the specimen. This feature removes the necessity for actual measurement of the scanned distance or the zero intercept.

However, accommodation of short fibers such as cotton was extremely difficult using the fiber scanner supplied with the instrument. According to a standard of the American Society for Testing and Materials[43] (which refers to the PPM-5R tester), the distance between transducers are required to be varied between 5 and 50 cm, an indirect indication of the minimum acceptable crystal separation. Hamburger[37] used 18 cm for the minimum separation, although he did not expect any difficulty in reducing the separation.

In the PPM-5R tester, it was found that the mounting of the transducer cases, and the size and orientation of the notches in the crystal tips, were unsuitable for specimens of short, fine fibers. In order to test short specimens having a pulse velocity of, say, 3 km/sec, the lowest of the PPM-5R microsecond range of 50 μs was clearly inadequate.

After a series of minor modifications to the existing apparatus had been attempted, it was decided that a completely new fiber scanner be constructed to suit the present requirements.

Problems Associated with the Scanner Design

The following three problems were encountered in relation to the design of an improved scanning device.

Accommodation of Short Specimens

The minimum distance between the tips of the two crystals is limited, not only by the effective length of the specimen but also, by the dimension of the transducer case in which the crystal must be suitably mounted, acoustically insulated, and protected mechanically.

Satisfactory Crystal–Fiber Contact

In order to introduce pulses of sufficient magnitude above noise level, the crystal–specimen contact must be adequate and steady. The only way to achieve satisfactory contact with existing instruments is to increase the lateral pressure between crystal and specimen; but in so doing, unintentional strains are imposed to the fiber specimen. The effect of such strain increments on the measured value of modulus may be considerable, especially at a low level of controlled fiber strain. Therefore, an improved crystal–specimen contact must be achieved without introducing unwanted fiber strain.

Maintenance of Constant Fiber Strain Level

The solution to the above-stated problems was found to introduce a new problem: the fiber strain level and the angle of the crystal–fiber contact vary as the fiber is scanned by the crystal. Any variation in either of these two quantities would practically nullify an otherwise precise determination of the dynamic modulus of the fiber as a function of the fiber strain level.

Solutions to the Above Problems

The first problem was solved by orienting the tips of the two crystals such that they faced each other, thus reducing the minimum practicable distance for crystal separation; and further, by mounting the fiber between a pair of pointed two-leaf clamps. In order to overcome additional difficulties arising from the shortness and fineness of the specimen, precision rack-and-pinion mechanical movements were utilized wherever required. A very fine slit was made in each piezo-electric crystal at its tip in order to accommodate fibers of small cross section, corresponding to less than 20 microns in diameter.

The second problem, relating to the crystal–specimen contact, was solved by employing "form constraint" rather than the more obvious "force constraint." The form constraint provides an effective contact between the specimen and the crystal, even at a low fiber strain level. The specimen path between the two clamps is made up of three linear segments. Both ends of the vertical center segment are in contact with the crystal tips, and the center segment makes a small angle to the two end segments of the specimen path. This arrangement permits much improved crystal–fiber contact through an increased, "wrap around" contact area.

A solution of the third problem was achieved by a combination of two modified slider-crank mechanisms with a specific crack arm ratio, determined by the mechanical properties of the fiber specimen under test. Figure 1 shows a schematic diagram of the kinematic relationship under-

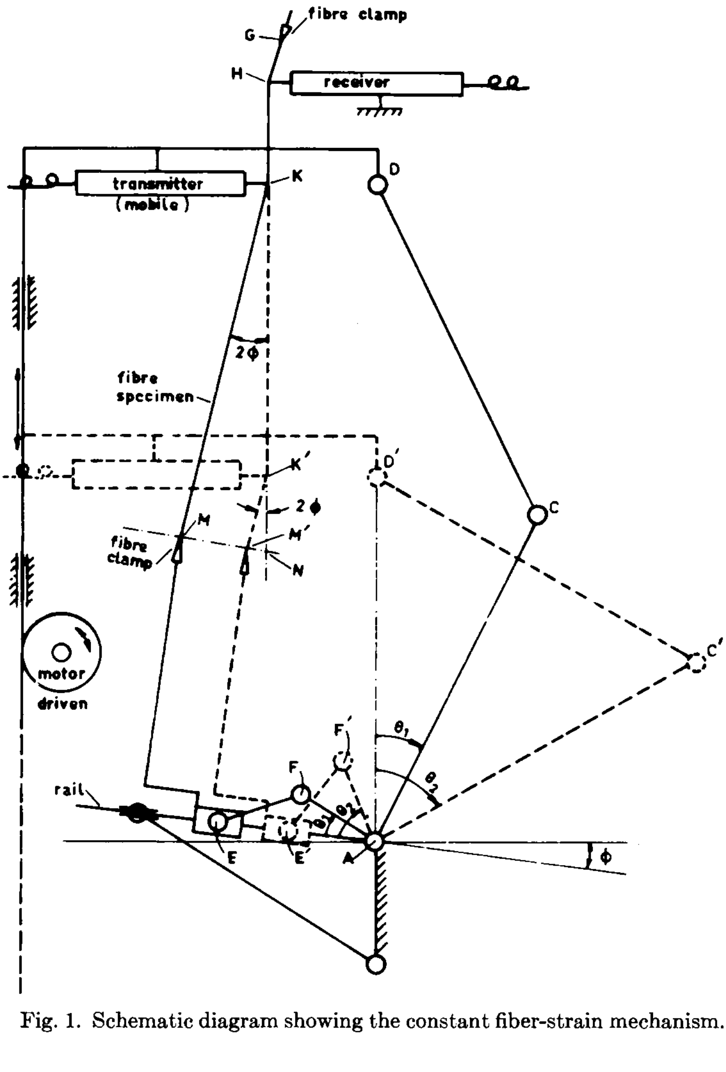


Fig. 1. Schematic diagram showing the constant fiber-strain mechanism.

lying the constant fiber strain and constant contact angle solutions. In the figure, the requirements for the constancy of fiber strain and fiber–crystal contact are

$$\text{path length GHKM} = \text{path GHK'M'}$$

$$\text{angle MKN} = \text{angle M'K'N}$$

These are equivalent to the requirement that the triangle MKN be isosceles and that

$$\begin{aligned} \text{length KM} &= \text{length KN} \\ &= \text{length KK}' + \text{K}'\text{M}' \\ \text{angle MKN} &= \text{angle M}'\text{K}'\text{N} \\ &= 2\,\phi \end{aligned}$$

where ϕ is the angle between the slider rail and the horizontal.

It has been found that the actual requirements for the mechanism are

$$\text{angle EAF} = \text{angle DAC} = \theta_1$$

$$\text{angle E}'\text{AF}' = \text{angle D}'\text{AC}' = \theta_2$$

and

$$\frac{\text{length of arm AF}}{\text{length of arm AC}} = 2 \sin \phi$$

where θ_1 and θ_2 are shown in Figure 1.

The angle ϕ, which determines the tilt of the sliding rail, is fixed by the required contact angle between the crystal and the fiber; this contact angle is dictated by the properties of the specimen. Once the angle ϕ is selected, the crank arms AF and AC are tightened to the crank shaft A. Suitable calibration tools have been prepared to ensure these kinematic conditions.

Other Modifications and Accessories

In order to obtain a wide traverse of the recording pen across the chart, the lowest PPM-5R microsecond range of 50 μs has been extended to 25 μs.

A simple reliable device was designed to accurately stretch the fiber at a given rate by a predetermined percentage strain of the order of 0.02%. The scanning and the determination of the modulus were conducted during an idle period between successive straining periods.

Experimental Techniques

A pair of fiber-holding compasses was constructed by attaching two small pairs of spring-loaded tweezers to a pair of drafting compasses. The separation between the tips of the normally closed tweezers were brought to less than the gauge length, say, 25 mm, to be used. After both ends of the single-fiber specimen were gripped with the tips of the tweezers on the compass, the tweezer tip separation was gradually increased while the fiber was viewed under a magnifier. This operation was carried out under bright illumination on a background of a black velvet-covered board in order to remove as much as possible of the existing crimps or kinks in the specimen, without actually prestraining the fiber.

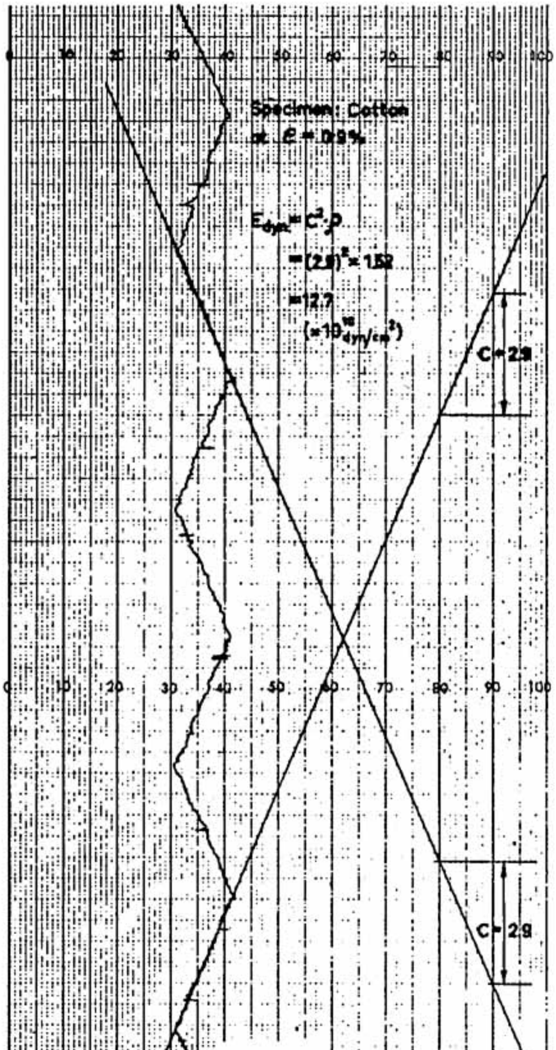


Fig. 2. An example of the actual relationship between distance scanned and transit time as plotted on the recorder chart.

The fiber-mounting pads have been prepared by punching out small sections of a folded thin cardboard sheet with a band of double-sided adhesive tape between them. The mounting pad was designed with an opening containing two pointed tips facing each other at a set gauge length.

The specimen, held by the fiber-holding compasses, was then transferred to the mounting pad. The mounting pad with the fiber attached was easily accommodated between two pairs of two-leaf spring clamps on the fiber scanner. The portions of the mounting pad that were not held under the clamps were cut away with a razor blade or a pair of nail clippers.

The velocity of propagation of the pulse in the specimen is given as the slope of the recorded lines, indicating the relationship between scan distance and the transit time. Figure 2 is an example of the actual record of the measurement for a cotton fiber with a gauge length of 25 mm.

In order to identify the zero strain level, i.e., the initial measurement during a fiber test, the relative positions of all moving parts were made measurable through the use of micromovements with vernier scales. A satisfactory way of identifying the zero strain level was found to be the combined use of these scales and the observation of the velocity of sound waves in the air across the crystal separation. When a transit time corresponding to the velocity of sound in the air is observed, the upper fiber clamp was raised by a minute amount using a fine adjustment, similar to the one found on a microscope, and a sudden jump of the recording pen was noted. This position was taken to be the zero fiber strain level for the test.

RESULTS AND DISCUSSION

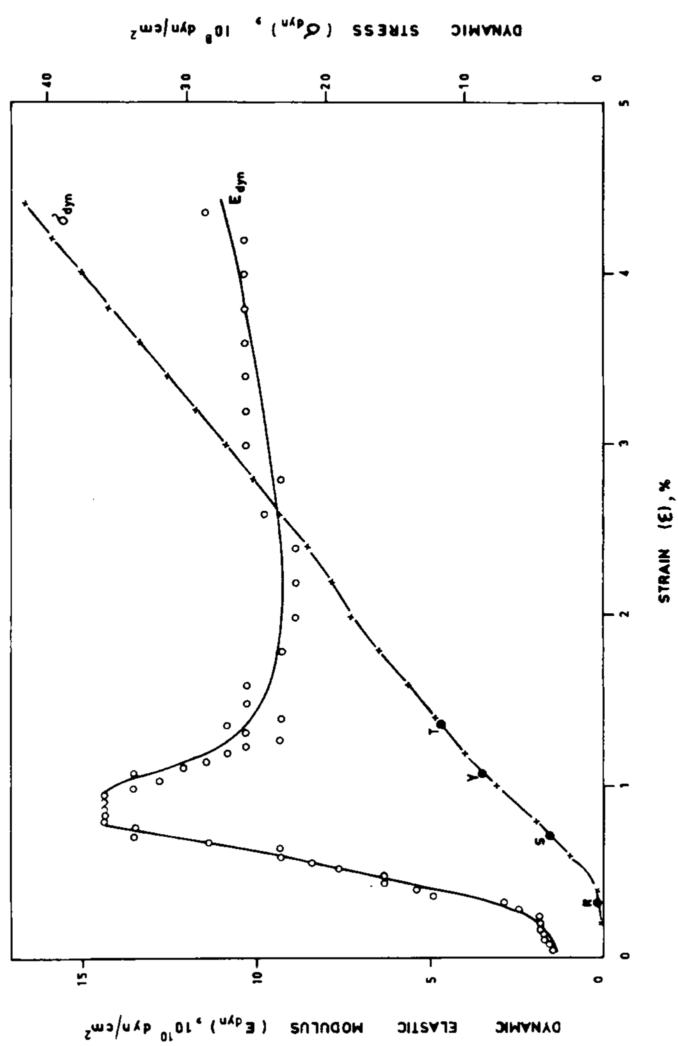
Figures 3 to 5 show several typical dynamic modulus–strain curves obtained for cotton, wool, and jute fibers. The dynamic stress–strain curves were constructed using the relationship

$$\sigma_{\mathrm{dyn}}(\epsilon) = \int_0^\epsilon E_{\mathrm{dyn}}(\epsilon)d\epsilon.$$

This method was used for rubber specimens by Lensky and Tarasova.[44]

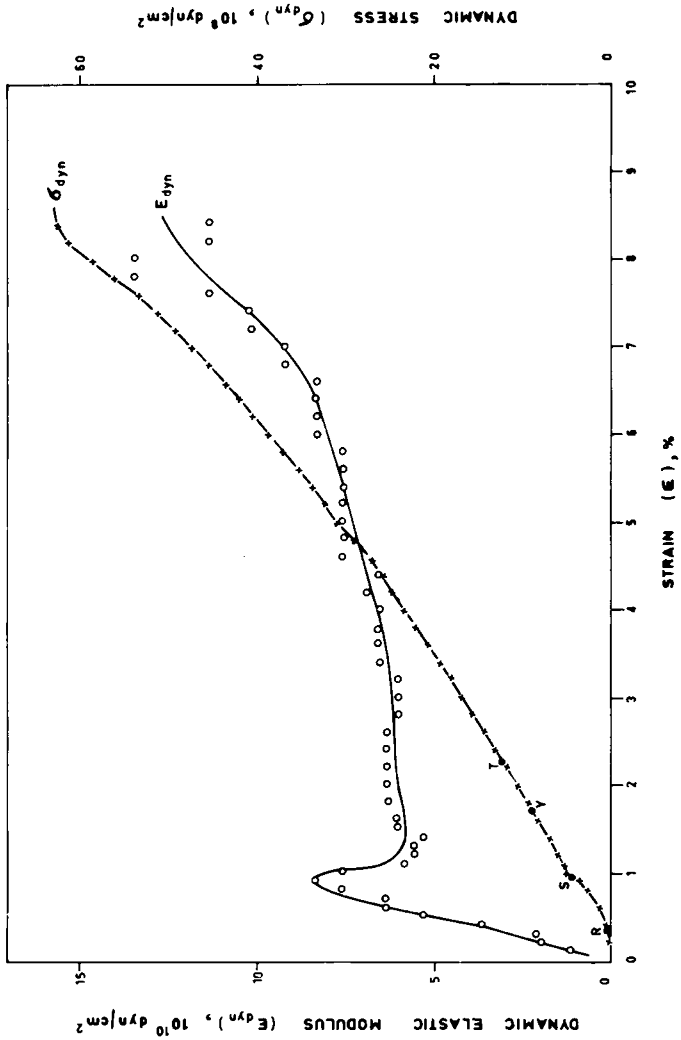
For all fibers tested, the dynamic modulus–strain curves appear to conform to certain general features as represented by the curve OF_m in Figure 6. The curve OF_s is the dynamic stress–strain curve constructed from the general curve OF_m.

In the general dynamic modulus–strain curve, it is apparent that the dynamic modulus assumes an initial peak value which signifies the onset of the yield phenomenon. It is clear that the initial or preyield region OB_s of the general dynamic stress–strain curve is concave toward the stress axis. The yield region of the curve is accompanied by a declining modulus and occurs between the two points of inflexion, B_s and D_s, on the dynamic stress–strain curve (corresponding, respectively, to maximum and minimum stationary values of the dynamic modulus). Another important aspect to emerge from a comparison of the individual curves for the different fibers is that the dynamic modulus again increases after the yield region, implying a region similar to strain-hardening (the prefracture or postyield region shown in Fig. 6). This last region is particularly evident in the curves for cotton B and wool B, where the dynamic modulus in the postyield region is seen to climb well beyond the previous peak level at the point B_m. The curves shown for cotton C and D shown in Figure 3 exhibit



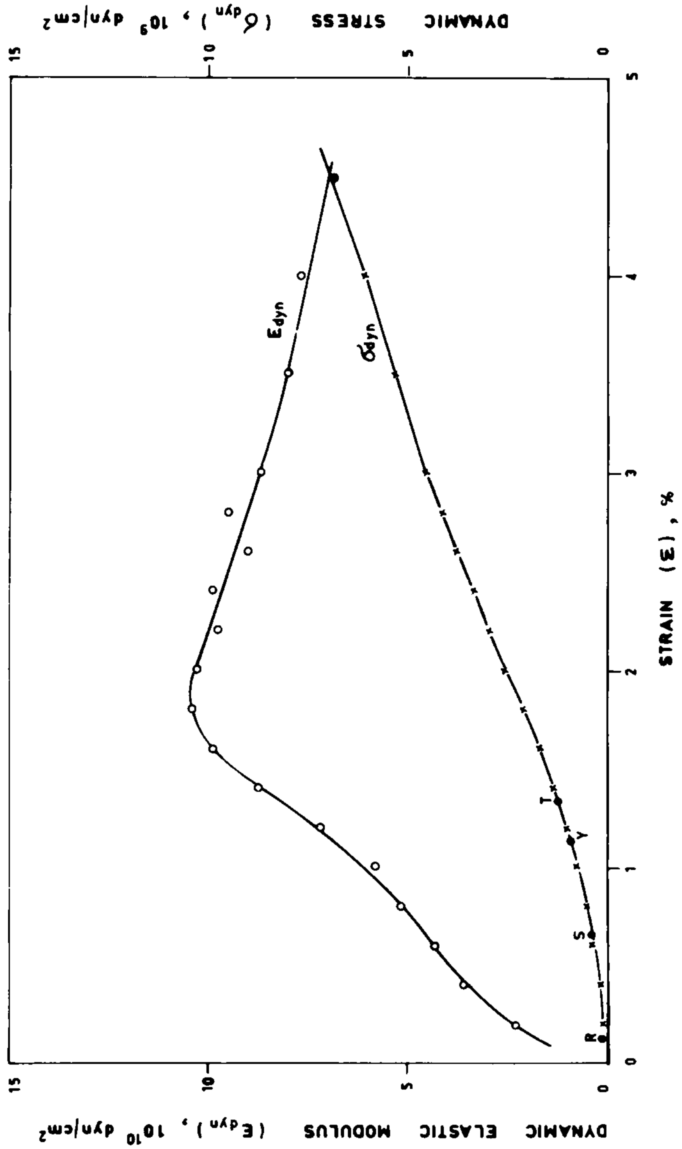
(a)

Fig. 3 (*continued*)



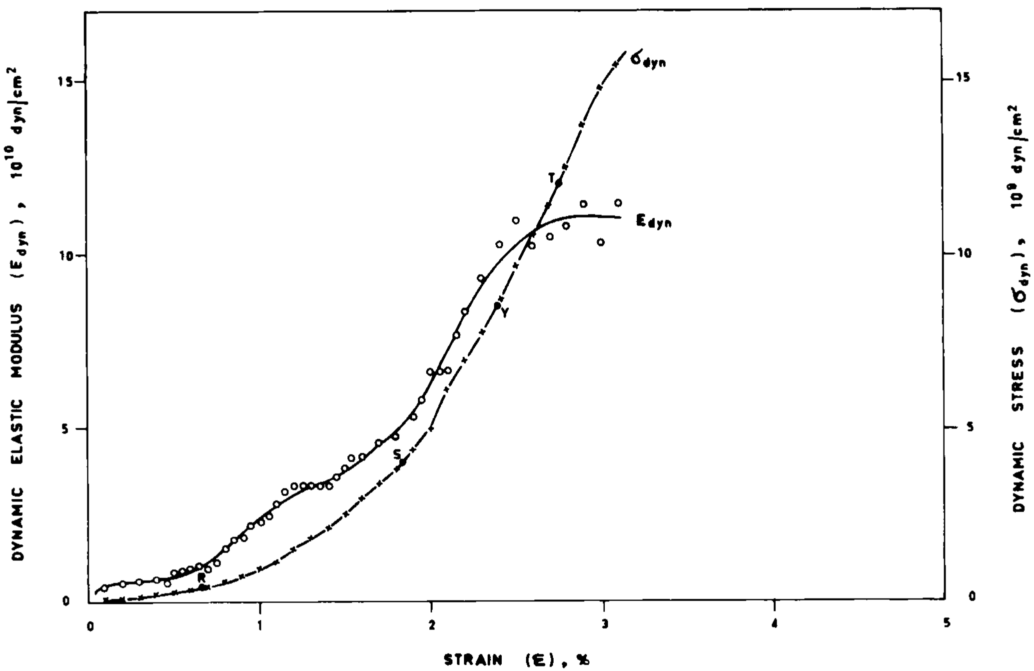
(b)

Fig. 3 *(continued)*



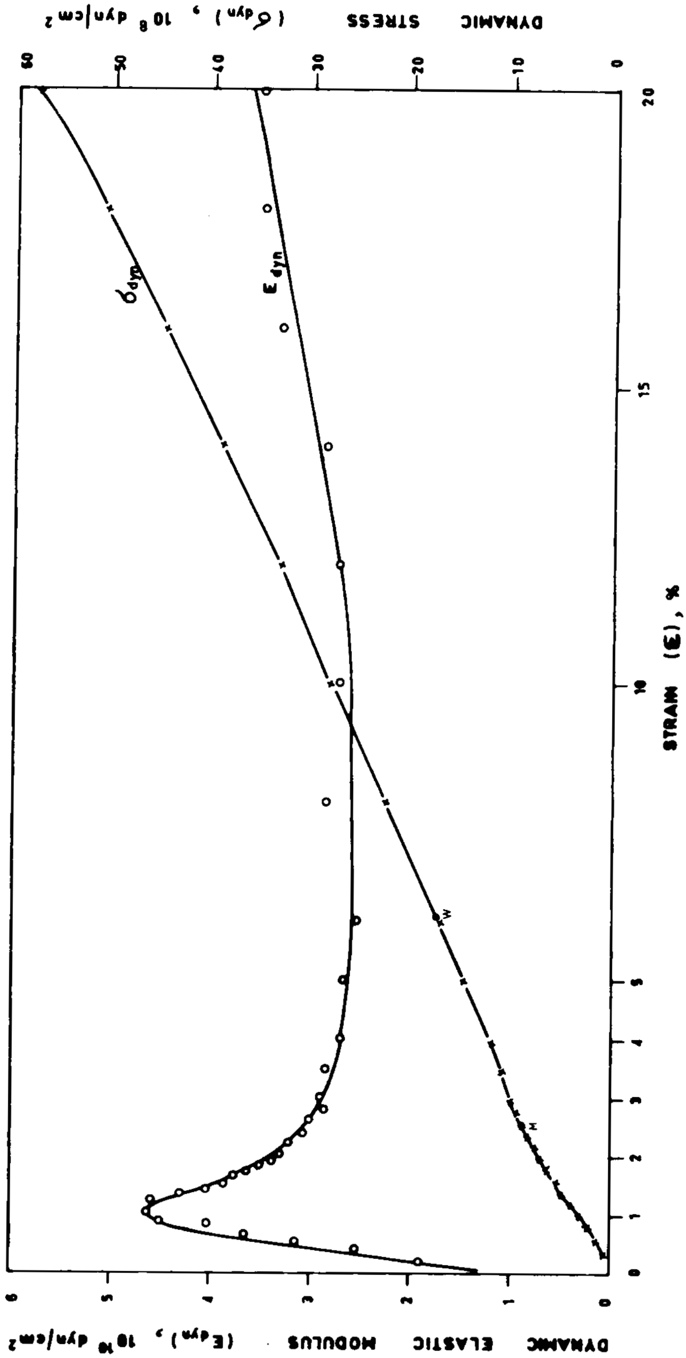
(c)

Fig. 3 (*continued*)



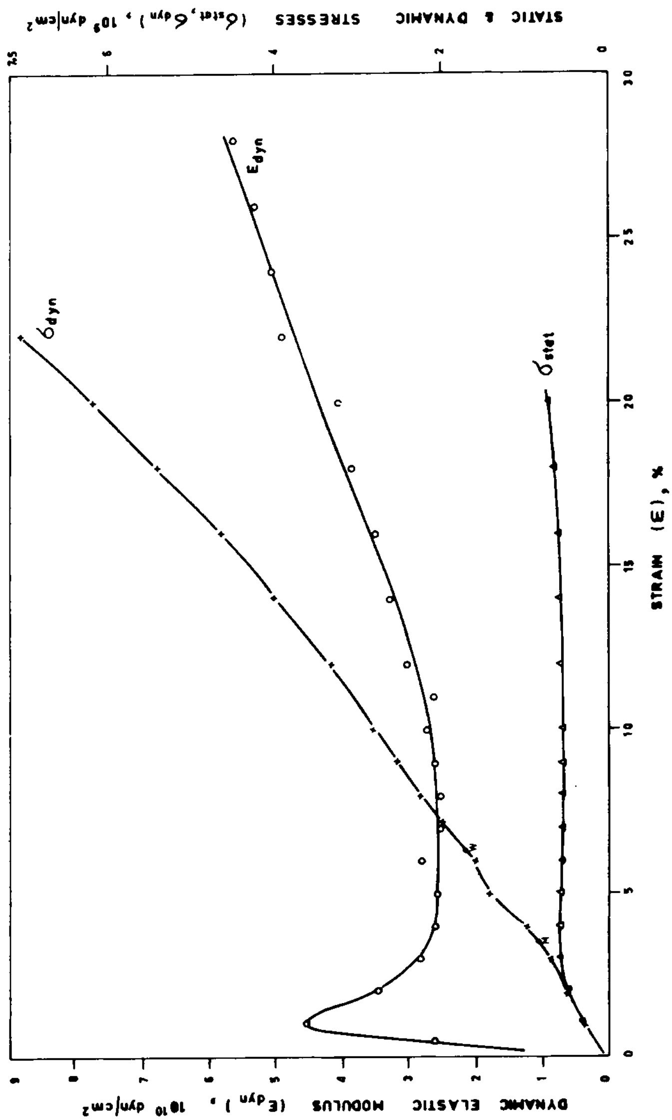
(d)

Fig. 3. *Measured* dynamic elastic modulus–strain curve and *constructed* dynamic stress–strain curve for: (a) cotton A (Deltapine variety); (b) cotton B (Deltapine variety); (c) cotton C (Indian Oomras, high maturity); (d) cotton D (an American cotton of low micronaire value).



(a)

Fig. 4 (*continued*)



(b)

Fig. 4. *Measured* dynamic elastic modulus–strain curve and *constructed* dynamic stress–strain curve for: (a) wool A (Merino 64's quality, 22 μ diameter); (b) wool B Lincoln, 27 μ diameter).

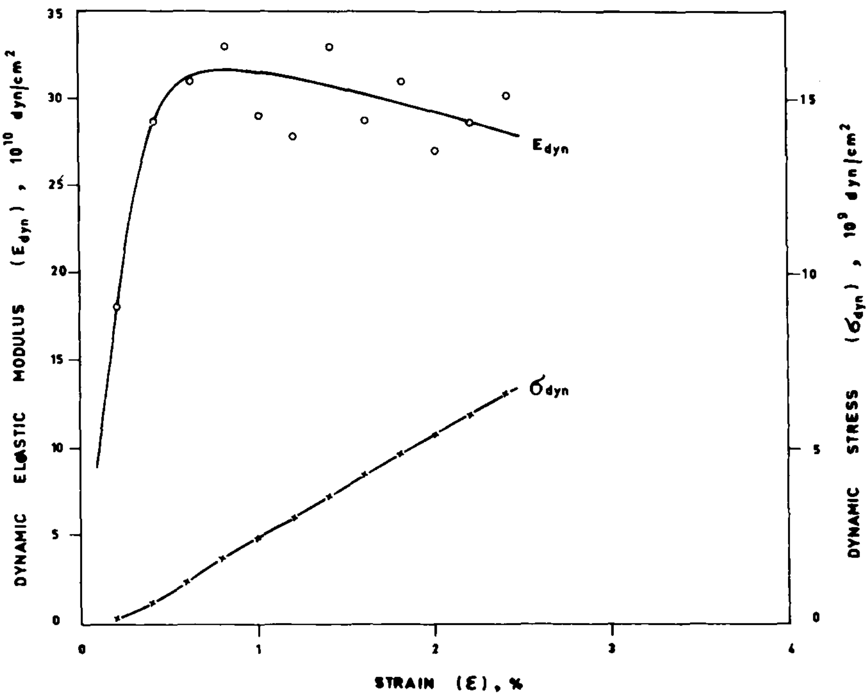


Fig. 5. *Measured* dynamic elastic modulus–strain curve and *constructed* dynamic stress–strain curve for jute.

the largest observed deviations from the shape of the general curves illustrated in Figure 6.

Published values of the dynamic elastic modulus listed in Table I for cotton and wool fibers seem to represent the yield or postyield values of the dynamic elastic modulus for the respective fiber types. In the case of the jute fibers tested, the modulus failed to reach the value of 66×10^{10} dyn/cm^2 for jute fibers given by Meyer and Lotmar.[33]

It is reasonable that the modulus values determined by other workers without means to control the fiber strain level should correspond to the yield or postyield region of the dynamic stress–strain curve.

It is evident from Figures 3 to 5 that the modulus values vary considerably as the fiber strain level is altered, indicating the importance of controlling the fiber strain level in these measurements.

Cotton Fibers

The curves for single cotton fibers are given in Figure 3: samples A and B are both of a Deltapine variety, C is an Indian Oomras cotton with a high maturity, and D is an American cotton of low micronaire value. In the present work, it was not intended to study in detail the effects of variety, maturity, or other factors on the measured results. However, it is clear from the curves that the method used is capable of revealing certain

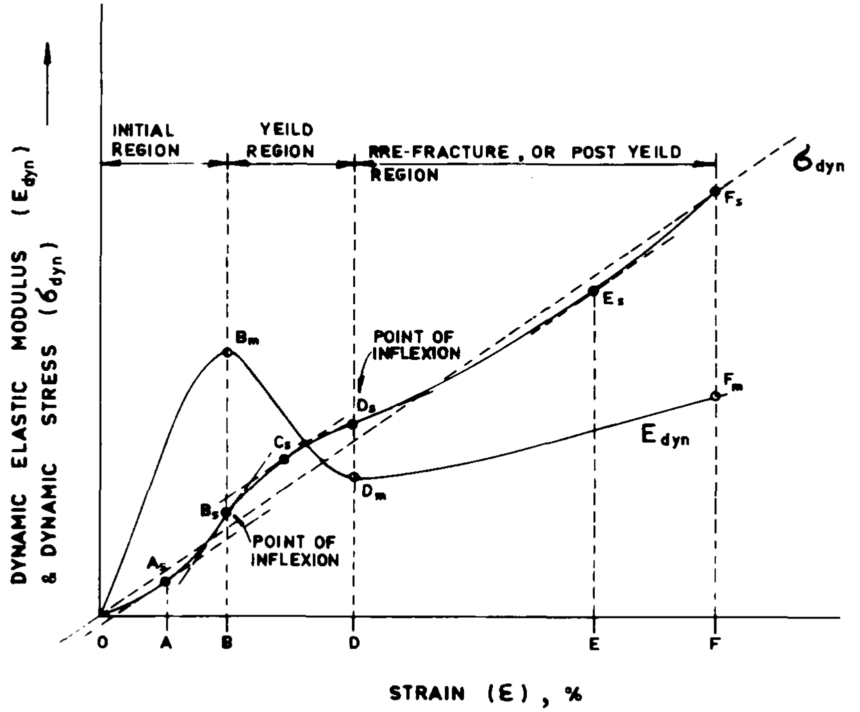


Fig. 6. Curves showing the general features of the fiber modulus–strain and dynamic stress–strain relationships.

features of the stress–strain behavior that are not evident using conventional methods.

According to Meredith,[45] different cotton fibers give similar (static or quasi-static) stress–strain curves, consisting of a linear portion (after the crimp has been removed by a stress of 0.03 g/den.), then a transition region at about 0.3 to 0.9 g/den., before the curve assumes a definite concavity to the stress axis (σ_{dyn} in 10^8 dyn/cm^2 = (σ_{dyn} in g/den) 8.84 × specific gravity). Wakeham[46] gives an average yield stress of 0.64 g/den. for cotton fibers. In order to facilitate comparison, the points corresponding to 0.03, 0.3, 0.64, and 0.9 g/den. have been marked R, S, Y, and T, respectively, on the constructed dynamic stress–strain curves for cotton fibers shown in Figure 3.

Except for cotton D, an early peak for the dynamic modulus–strain curve clearly demonstrates an initial concavity to the stress axis of the dynamic stress–strain curve. This concave portion extends to the first point of inflexion which corresponds to the peak modulus. For the dynamic stress–strain curves of cotton A and B, the point S is in the vicinity of this inflexion point. This initial concave region of the curve corresponds to a similar observation for Sakel cotton by Brown, Mann, and Peirce.[47]

Wool Fibers

Figure 4 gives the curves for single wool fibers: A is a Merino wool quality 64's (22 μ diameter) and B is a Lincoln wool of 27 μ diameter. In the case of the Lincoln wool, a single fiber was cut into four parts. The first and third parts were tested for dynamic modulus, while the second and fourth parts were tested on an Instron tester to obtain the (static) stress–strain curve. Figure 4 shows that the initial region of the static stress–strain curve is similar to the initial region of the constructed dynamic stress–strain curve. In most cases, wool fibers give dynamic curves similar in shape to those of cotton A and B, except that the wool fibers exhibited lower dynamic modulus and larger breaking strain.

It is difficult to define precisely the yield point on a static stress–strain curve, especially for a wool fiber which manifests a gradual flattening of the curve in the yield region. Also, the actual physical significance of the yield point may be argued in such cases.

Meredith[45] gives a yield stress of 0.64 g/den. for Botany 64's wool, and Woods[48] gives a value of 1.26 g/den. for keratin. These values are marked M and W, respectively, on the constructed dynamic stress–strain curves in Figure 4. It is interesting to note that, for both wool A and B, the point M is approximately midway between the two inflexion points and the point W almost coincides with the second inflexion point.

Jute Fibers

Figure 5 gives an example of the curves for a jute fiber. Relatively few jute fibers have been tested, but very similar features were reproduced in all tests. Although the scatter of the observed points in the latter half of the curve is greater than for the other fibers, the curve appears to be a truncated form of the curves obtained for cotton and wool fibers. It should be noted in this respect that dynamic modulus of jute fibers is considerably higher, and the breaking strain is considerably lower than for the other fibers tested.

General Fiber Dynamic Stress–Strain Curve

A set of model curves, depicting the general features of the measured dynamic modulus–strain curves and the constructed dynamic stress–strain curves, is presented in Figure 6. In the figure, the point B has the following properties:

$$\sigma''_{\text{dyn}}(\text{B}) = E'_{\text{dyn}}(\text{B}) = 0$$

$$\sigma'''_{\text{dyn}}(\text{B}) = E''_{\text{dyn}}(\text{B}) < 0$$

where

$$\sigma_{\text{dyn}}^{(n)}(\text{B}) = (d^n\sigma_{\text{dyn}}/d\epsilon^n)_{\epsilon=\text{B}}.$$

It follows the the point B_s is an inflexion point on the constructed dynamic stress–strain curve, OF_s. Likewise, the point D_s is the second point of inflexion on the same curve.

The three points A_s, C_s, and E_s are those points where the tangents to the curve are parallel to the line joining the origin 0 and the breaking point F_s. This was the requisite condition, adopted by Meredith,[45] for a yield point. Beyond the yield point so defined, the fiber should extend at a greater-than-average rate. Of the three points, only C_s fits the requirements cited by Meredith; beyond the points A_s and E_s, the modulus increases, and the fiber extends at a lower rate than average.

However, it is clear from the figure that the actual fall in the dynamic modulus begins at the point B and continues until the point D is reached. Therefore, the point C_s cannot signify a sudden drop in modulus or a sudden increase in extension for a small increase in stress. Indeed, the exact definition of the yield point is particularly difficult in the case of a polymeric fiber. Meredith notes that, if a fiber is stretched beyond its yield point, it does not show complete recovery, although it may creep back slowly to its original length. Alternatively, the yield point is often assumed to be the point at which the relationship between stress and strain departs from linearity. The results of the present work have shown that linearity does not always occur for the preyield region of the dynamic stress–strain curve, although the portion 0-A_s-B_s of the curve for wool fibers is very close to linear.

Following the accepted practice of dividing a stress–strain curve into three regions, the initial, yield, and the prefracture or postyield regions have been marked on the general curve of Figure 6. In the figure, it is seen that these regions may be specified by the two inflexion points on the constructed dynamic stress–strain curves without ambiguity.

CONCLUSIONS

A comparison of experimental techniques available for the measurement of the dynamic elastic modulus of short staple fibers has indicated that the acoustic pulse propagation method appears to be the most convenient principle for this application. An instrument based on this principle has been designed in which the dynamic elastic modulus is measured at varying levels of fiber strain. One of the principal features of this technique is that the measurement is independent of the specimen dimensions.

Experimental results for cotton, wool, and jute fibers were used to construct the dynamic stress–strain curves for these fibers. For the constructed dynamic stress–strain curves, two points of inflexion were determined, without ambiguity, by the maximum and minimum values of the fiber dynamic elastic modulus.

The yield region for the fiber specimen is clearly specified as the region between the two inflexion points. It was also shown that the preyield region, beyond the "decrimping" stage, is generally not linear but rather concave toward the stress axis. Large differences in the dynamic elastic modulus and fiber breaking strain were noted for the three different fiber types tested.

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